View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Queensland University of Technology ePrints Archive

Queensland University of Technology
Brisbane Australia

This is the author's version of a work that was submitted/accepted for publication in the following source:

Quinnell, Lorna & Carter, Merilyn
(2013)
Gibberish or what? Use of symbolic language in primary mathematics.
Australian Primary Mathematics Classroom, *18*(1), pp. 8-14.

This file was downloaded from: https://eprints.qut.edu.au/106716/

© 2013 Australian Association of Mathematics Teachers

Notice: *Changes introduced as a result of publishing processes such as copy-editing and formatting may not be reflected in this document. For a definitive version of this work, please refer to the published source:*

https://search.informit.org/documentSummary;dn=395561902855204;res=IELHSS

Gibberish or what? Use of mathematical symbols and abbreviations in primary years

Authors

1. Lorna Quinnell

Lecturer, School of Education

Faculty of Education, Health & Science

Charles Darwin University

Casuarina Campus, Darwin, Northern Territory

Email: lorna.quinnell@cdu.edu.au

2. Merilyn Carter

Numeracy Consultant, Count on Numeracy

22 Grayson St

Morningside

Queensland, 4170, Australia

Email: countonnumeracy@bigpond.com

Gibberish or what? Use of mathematical symbols and abbreviations in primary years

Key words: Mathematics, Symbols, Abbreviations, Language, NAPLAN

Abstract

Symbols and abbreviations are commonplace in mathematics text. In this article the use of mathematical symbols and abbreviations will be discussed with reference to examples from recent NAPLAN tests. The article will draw the reader's attention to the large variety of symbols and abbreviations used, with particular focus on primary years. Thereafter, the difficulties which learners may experience because of this unique and complex part of the mathematics register will be discussed. Knowledge of the complexities which symbols and abbreviations introduce into mathematics can enable a teacher to pre-empt misunderstandings and aid learners in this area of mathematics.

Gibberish or what? Use of mathematical symbols and abbreviations in primary years

Symbols, abbreviations, and conventions are commonplace in mathematics text, allowing mathematical ideas to be communicated concisely (Bobis, Mulligan & Lowrie, 2004). Knowledge of the complexities that they introduce into mathematics can enable a teacher to pre-empt misunderstandings and aid learners in this area of mathematics (Rubenstein & Thompson, 2001). This article draws the reader’s attention to the large variety of symbols, abbreviations, and conventions used in mathematics, with particular focus on primary years. Thereafter, the difficulties that learners may experience because of this unique and complex part of the mathematics register are discussed. The article makes use of examples from 2010 National Assessment Program: Literacy and Numeracy (NAPLAN) tests. However the discussion applies to the broader use of symbols and abbreviations in mathematics classrooms and in everyday life. As will be noted in this article, the use of symbols and abbreviations are controlled by very specific guidelines such as those found in the Australian Style Guide (Commonwealth of Australia, 2002), which occasionally differ from those used in NAPLAN testing.

Use of symbols and abbreviations in the 2010 NAPLAN numeracy tests

Symbols and abbreviations used in mathematics are varied and increasingly more use is made of symbols in higher levels of schooling. This is evident in NAPLAN numeracy tests. For instance in the 2010 NAPLAN numeracy tests (ACARA, 2010), symbols for the four operations +, -, ÷, and ×, as well as units such as m and cm and other symbols such as $ and = are used in year 3. In addition to the above, in year 5 the decimal point, fractions such as $15 \div 3$ or $\frac{15}{3}$, units such as g, kg, ml, symbols such as >, N for north, C4 for a grid reference and 3D for three dimensional are also used. In year 7 and 9 units such as km, mm, mg, m^2, cm^2 are introduced, as well as times such as 3:20, symbols such as ( ), %, √, and exponents such as 10^3, negative numbers such as -18°C, ratios such as 1:4, intervals such as 145-149, 5|2 meaning 52, abbreviations such as am, pm, and min for minute, and variables such as *x* and *y*.

It must be noted that sometimes numbers and units written in words or a combination of numbers and words are used in NAPLAN test questions. Examples in the NAPLAN tests are the words *half* and *two and a half* in year 3 and *centimetre* and *square metres* in year 5. For example, the Year 3, 2010 NAPLAN numeracy paper, question 25 asked: “How many quarters are there in two and a half oranges” (ACARA, 2010, p. 12). A combination of words and a number is evident in statements such as the following found in question 12 in the Year 7, non-calculator, 2010 numeracy paper: “A flea can jump up to 200 times its body length” (ACARA, 2010, p. 5).

Categories of symbols and abbreviations

The symbols and abbreviations used on primary NAPLAN numeracy tests (ACARA, 2010) can be categorised into five different groups.

- numbers including fractions and decimals
- mathematical operations such as +, -, × or ÷
- comparatives such as = or ≤
- other symbols, such as tallies and colons in tables, symbols for SI units such g for gram, and $ for dollar and conventions such as N for north
- mixed symbols such as C4, 3D

The variety and complexity of the symbolic language of mathematics is an indication of the linguistic complications which they may introduce into mathematics. A discussion of areas of possible confusion are discussed below.

Difficulties posed by the use of symbols and abbreviations in mathematics

The use of mathematical symbols poses challenges to learners in terms of reading, writing, and verbalising symbols (Rubenstein & Thompson, 2001). Difficulties are related to both syntax and semantics. The syntax includes factors such as verbalisation and spelling whereas the semantics refers to the underlying meaning of the symbol or abbreviation. Thus in the case of 2^3 the student needs to be able to verbalise and spell "two to the power three" but also needs an understanding of the meaning of 2^3. Challenges posed by the reading and writing of mathematical symbols and abbreviations are discussed below.

Symbols for number

At the very foundations of mathematics, the symbols for low Arabic numbers give no indication of the size of the number. This is in contrast to the simpler Roman and Chinese number symbols for numbers one to four which have progressively more strokes. Arabic digits such as 4 and 7 can be written or printed in different ways. As is the case for other numbers, they can also be represented as Roman numerals and tallies such as those seen in the Year 3, 2010 NAPLAN numeracy paper, question 1. Further, a digit such as 4 can have many different meanings, which is evident in examples such as 40, 0.4, $\sqrt{4}$, 42, an understanding of which is fundamental to the understanding of mathematics concepts. Numbers are used in the cardinal, ordinal, or nominal sense. The cardinal numbers are most common and are counting numbers such as those used in calculations where the number 2 has a lower value than 6. Ordinal numbers are positional numbers such as 6th in a race. In this case, 2nd place is understood to be a higher position than 6th place.

Examples of nominal numbers are numbers used on rugby jerseys or post boxes. Both ordinal and nominal numbers are used in the Year 7, calculator, 2010 NAPLAN numeracy paper, question 8, which refers to the 1st house on one side of a road being number 2 and asks for the number of the 18th house.

Similar symbols with different meanings

In some situations, symbols or abbreviations have a similar appearance but very different meanings (Gough, 2007). Examples are: the symbol f r zero 0, the symbol for the letter O, and the symbol for degrees °; the symbol for multiplication ×, and the symbol for the letter x, and the variable *x* (italicised). The numeral 1 can be confused with the lower case letter l. Another example is the symbol for subtraction, the symbol used to represent a negative number, and the symbol used to represent an interval, in examples 7 – 5, –9, and pages 5–9, respectively. To avoid confusion use of 15°C to 25°C is recommended rather than 15–25°C, for instance (Commonwealth of Australia, 2002). The slash used in a date such as 7/2/89, in a unit such as metre per second sometimes abbreviated m/s, and in the fraction three-quarters sometimes written as ¾ or 3/4 is similar to the symbol used in a stem and leaf graph where 3|4 means 34. Note that the format of ¾ is seen as preferable to ¾ for a fraction (Commonwealth of Australia, 2002), and the horizontal vinculum is the preferred way to write fractions in NAPLAN tests. The dot for a decimal point or money can be confused with the dot often used to represent time such as 2.40 pm and the dot sometimes used in dates (Commonwealth of Australia, 2002). The colon used in ratios can be confused with the colon used in the recording of time. Unlike the Australian style guide, which uses a dot for time (Commonwealth of Australia, 2002), a colon is generally used to represent time in NAPLAN tests in all levels (e.g., 2:45 am). However in question 16 of the Year 9, non-calculator, 2010 numeracy paper both the colon (e.g., 11:00 am, Thursday) and the dot are used for time. The dot was used in the picture of a road sign limiting parking time for cars (1 HOUR PARKING 8.30AM–6.00PM MON–FRI). Note, too the difference between the use of uppercase and lowercase letters, font sizes, abbreviations for the days, and spaces before the am or pm. Use of AM, PM originates from the American English way of representing time (Commonwealth of Australia, 2002). The words noon and midnight can be used to avoid misunderstanding of the terms 12 am and 12 pm (Commonwealth of Australia, 2002). Abbreviations for days and months are usually the first three letters followed by a dot except for September (e.g., Mon. and Jan. and Sept.), unless space is limited (Commonwealth of Australia, 2002). In all the above cases, the context of the questions gives clues about the actual meaning and use of the symbolic language. The examples are summarised in Table 1 in which similar symbols are used in examples in columns A, B, and C. It must be noted that use of symbols in society does not always conform to standard conventions.

Table 1: *Confusing Symbols with Similar Appearances*

A	B	C
Zero 0	Uppercase letter O	
Multiplication ×	Lowercase letter x	Variable x
Number 1	Lowercase letter l	
Colon in ratios 2:5	Colon in times 2:45	
Subtraction 7 – 5	Negative number $^{-}9$	Interval 5-9
Key in stem and leaf graph 7|3	Fraction 7/3	

There are other cases of symbols and abbreviations being used differently in various contexts. For instance 3D can represent three dimensional, a grid reference on a map, or a question number. Use is made of the symbol 3D for three dimensional objects in question 14 of the Year 5, 2010 NAPLAN numeracy test and the grid reference C4 can be seen in question 10 of the same paper. The use of 154/4 and 3.2 overs in cricket have a totally different meaning to the mathematical meaning of 154/4 as a fraction (although not the preferred way of writing a fraction), and the decimal number 3.2 in mathematics.

Need for care with reading and writing mathematics

Mathematics text needs to be written and read very carefully as small differences in the use of symbols can cause major changes in meaning or value. For example, the numbers 2.617 and 2,617, the number 150 and the temperature 15°, the mathematical terms 2x and 2x, and the expressions $2 + 7 \times 3$ and the similar expression $(2 + 7) \times 3$ have entirely different meanings (values). To aid understanding, there are set conventions for spacing in examples such as those above. Thus, for instance, there are spaces before and after the + and symbols in $(2 + 7) \times 3$, no space in a number such as 2617, but spaces in bigger numbers such as 23 456 or 1 000 000 (Commonwealth of Australia, 2002). Correct use of symbols, abbreviations, and conventions is particularly important in engineering, architectural, and medical fields where careless use of the symbolic language of mathematics could result in serious consequences.

Implied meanings of symbols

Other factors add to the complexities of the use of symbols in mathematics. Sometimes symbols are invisible but implied (e.g., 2 implies 2 + ¾and 2x implies $2 \times x$ whereas in other cases there is no implied symbol e.g., $6).

Similar concepts represented by different symbols

In some cases, concepts can be represented in a number of ways, for instance, parts of a whole can be represented as fractions, ratios, decimals, and percentages (Lemke, 2003). This is evident in the NAPLAN Year 7, non-calculator, 2010 numeracy paper, question 21, which requires students to convert the fraction into the decimal 88%. In some cases different symbols have the same meaning, for example 2^3 can be represented by 2 ^ 3 on a calculator screen, and 9 ÷ 3, 9−3, 9/3 or 3 9 all represent nine divided by three.

From prose to symbols

Much has been written about the difficulties which learners can experience when mapping word problems into symbolic form. In the example, "nine divided by three" above, it becomes evident that the order of the words is sometimes different from the order of the symbols (Schleppegrell, 2007), which can cause reversal errors. For instance, a student who encounters the term "five less than a number" may write 5 – n rather than the correct form n – 5. The terms "divide 25 by 5," "divide 5 into 25," "divide 25 into 5 equal parts" have identical meanings although the word-order varies. Often students simply divide by the smaller number. Although order is crucial in some instances, in others it is not, for instance $48 \times 6 = 6 \times 48$ but $48 \div 6 \neq 6 \div 48$. Unlike prose, symbols and abbreviations are not always written in a line and read from left to right. Examples of such variations are found in units such as $6 for instance, and in computations where numbers may be placed below each other to be added. Many such examples can be found in NAPLAN tests. Transposing words into symbols can require specific use of not only order but also size and position of symbols (Schleppegrell, 2007). This is evident in the example in which the phrase "the sum of the squares of 6 and 7" is transformed into $6^2 + 7^2$. A number of words such as "greater than or equal to" can sometimes be represented by a single symbol ≥.

Often many words or phrases such as "add," "plus," and "sum" can be represented by the same symbol. There are at least 20 ways of expressing 9 – 3 which include "the difference between nine and three," "decrease nine by three," and "nine take away three" (NSW

Department of Education, 1989). A specific word does not necessarily match a particular mathematical symbol. Thus, directing children to key words has its pitfalls (NSW Department of School Education, 1997; Zevenbergen, Dole & Wright, 2004). For example, the word altogether "has been used in a multitude of questions in recent NAPLAN numeracy tests. Altogether is often taken to imply addition. However, this is not always the case which can be seen by perusing questions 12, 13, 27, 30, 33 in the Year 3, 2010 numeracy paper" (Quinnell, 2011, p. 19). Examples that make use of the word altogether, similar to those in the NAPLAN tests, can be viewed in Figure 1 below. Note that sometimes the word altogether is in bold print and at other times not, to allow for an analysis of the impact that this has on the learners in terms of answering the questions.

Example 1: Jemma puts books into 5 equal piles. Each pile has 12 books. How many books does Jemma have altogether?

Example 2: A shop has 10 boxes of pens. Six boxes have 4 pens in each box. The other boxes have 6 pens in each box. How many pens does the shop have altogether?

Figure 1. Use of the word 'altogether' in mathematics questions.

The process of converting a mathematics problem to symbols allows a problem to be concisely represented and divorces a problem from its context allowing the solution to be applied in many situations. However, the lack of context may make the mathematical process appear impractical, the solution of which appears to apply to nothing (Bobis et al., 2004). Transposing words into symbols may pose particular challenges in algebra. Radford and Puig (2007) argued that “a great number of important difficulties encountered in the learning of elementary algebra are related to students’ understanding of the meaning of signs and syntax of the algebraic language” (p. 146).

Abbreviations

In addition to the challenges posed by mathematical symbols, a multitude of abbreviations are used in mathematics. These include abbreviations for physical quantities such as length l and symbols for the units attached to these quantities such as metre, m. However the l for length can be confused with the numeral 1 and the m for metre can also be used to represent the physical quantity mass, the word million (e.g., $3m), the prefix milli-, or if italicized, it can represent a gradient in higher levels. Mostly lower case letters are used, exceptions being N, S, E, and W for the four compass directions; 3D for three dimensional; and C4 for a grid reference. In terms of units, lower case letters are used, with an exception of units which originate from peoples’ names such as Watt, W; Joule, J; and others such as mega, M (to prevent confusion with the lower case m in milli); and litre, L (to prevent confusion with the numeral 1) (Commonwealth of Australia, 2002). Both abbreviations mL and L can be seen in question 40 of the Year 5, 2010 numeracy paper. Other units used in the same paper are m, cm, g and kg. Unit plurals have the same symbol/abbreviations as the singular form, for example 5 mm not 5 mms with a space between the number and symbol except in the case of angles such as 25°

(Commonwealth of Australia, 2002). “Most units of measurement used in Australia are those of the International System of units” (Commonwealth of Australia, 2002, p. 178), referred to as SI units. Although units such as ft for foot, lb for pound, and mi for mile are found in some textbooks and websites, they are no longer officially used in Australia. Units of time are not part of the SI system of units. Various abbreviations are seen for these units in different texts but they are officially s for second, min for minute, h for hour and d for day (Commonwealth of Australia, 2002).

A combination of digits, decimals and SI units are visible in question 22 of the Year 7, non-calculator, 2010 NAPLAN numeracy paper which asks students to identify the longest distance between 0.1203 km, 123 m, 1230 cm, and 12 030 mm. This is an example of some of the challenges posed to students by the symbols and abbreviations in mathematics.

Variations in different countries

Children from foreign countries may be unfamiliar with certain of the symbols and abbreviations used in Australia. In countries such as Australia and USA, the decimal point is represented by a dot and spaces are used to segment large numbers such as 2 267 489 (commas are still sometimes seen in documents which represent large numbers such as 2,267,489). In countries such as the European countries and South Africa, a comma is used to represent a decimal point. Some symbols vary from country to country (Rubenstein & Thompson, 2001), such as the Mexican symbol used for the number nine which is similar to a lower case g (Van De Walle, Karp & Bay-Williams, 2010). Nonmetric units are still used in some countries. Symbols for money vary with $ for dollars and cents used in Australia (e.g., 8c or $0.08, always written with two decimal places), £ for pound used in England, and € for euro used in Europe, for instance. To distinguish between dollars in different countries, abbreviations such as AU$25, NZ$25, and US$25 are used. Also possibly unfamiliar to children from non- English speaking countries are symbols such as am and pm, S for south, and 3D for three dimensional. In Holland, for instance, south south east (SSE) is abbreviated ZZO for zuid zuid oost and no equivalent abbreviations for am and pm are used. An understanding of such sources of confusion is important for teachers who may be teaching children from other countries and accessing resources from foreign websites and textbooks.

Some teaching ideas

The above examples draw attention to the barriers which mathematical symbols and abbreviations can pose to learners of mathematics. Since the symbolic language of mathematics represents mathematical ideas, the ideas need to be investigated prior to the introduction of the symbolic language which communicates the ideas (Booker, Bond, Sparrow & Swan, 2004; Rubenstein & Thompson, 2001). Symbols and abbreviations need to be reviewed which can be aided by the construction of a chart containing symbols, abbreviations, and the associated words (Rubenstein & Thompson, 2001). Such lists can be displayed in the classroom, used as a place mat on students' desks, or used to design a teaching game similar to dominoes. Teaching needs to emphasise visualising or understanding of concepts (Boulet, 2007), for instance the abbreviation m^2 needs to be associated with the visual understanding of the concept. It also needs to draw attention to possible misunderstandings and emphasise correct writing of symbols in terms of size, order, and positioning (Rubenstein & Thompson, 2001). For example, superscripts and subscripts need to be carefully written. Correct verbalisation of symbols can aid understanding (Rubenstein & Thompson, 2001). For instance, reading 0.4 as "fourteenths," 3^2 as "three squared" not "three to the two," 5 m^2 as "five square metres" not "five metres squared," and $x \leq 6$ as "x is less than or equal to six" not "x is less than and equal to six," is a bridge to understanding.

The following are some examples of activities which incorporate practice with symbols and make links between the different semiotic systems of mathematics such as written, symbolic and oral (Rubenstein & Thompson, 2001):

- students adding missing symbols to statements to make the statement true;
- pairs of students practicing reading and writing symbolic statements;
- students drawing examples and counter examples of symbolic statements such as $\overline{AB} = \overline{XY}$, $\overline{AB} \perp \overline{XY}$;
- students writing symbolic statements which correspond to mathematical diagrams.

In order to prevent student confusion, teachers need to reproduce symbols and abbreviations correctly in teacher-generated materials, bearing in mind the conventions referred to in this article and in the style manual (Commonwealth of Australia, 2002).

Conclusions

The use of symbols and abbreviations adds linguistic complexity to mathematical text and learners need to learn how to use this unique part of the mathematical register successfully. In particular, the transformation of mathematics word problems into symbolic form can pose difficulties to students.

"Focusing on the use of language is a crucial strategy in good mathematics teaching and a teacher's guidance can assist students to master the language of mathematics" (Quinnell &

Carter, 2011, p. 49). This includes a focus on the symbolic language of mathematics. In order to ease the linguistic load for mathematics learners, teachers need a comprehension of the variety of symbols and abbreviations and the difficulties which they pose to learners.

References

Australian Curriculum Assessment and Reporting Authority (ACARA). (2010). *National assessment program literacy and numeracy*. Retrieved 10 March 2011 from http://www.naplan.edu.au/tests/tests_landing_page.html

Bobis, J., Mulligan, J., & Lowrie, T. (2004). *Mathematics for children: Challenging children to think mathematically* (2nd ed.). Frenchs Forest, NSW: Pearson Education Australia.

Booker, G., Bond, D., Sparrow, L., & Swan, P. (2004). *Teaching primary mathematics* (3rd ed.). Frenchs Forest, NSW: Pearson Education Australia.

Boulet, G. (2007). How does language impact the learning of mathematics? Let me count the ways. *Journal of Teaching and Learning*, 5(1), 1–12.

Commonwealth of Australia. (2002). *Style manual: For authors, editors and printers* (6th ed., rev. Snooks &Co.). Canberra: John Wiley & Sons.

Gough, J. (2007). Conceptual complexity and apparent contradictions in mathematics language. *The Australian Mathematics Teacher*, 63(2), 8–15.

Lemke, J. L. (2003). Mathematics in the middle: Measure, picture, gesture, sign, and word. In M.Anderson, A. Sáenz-Ludlow, S. Zellweger & V. V. Cifarelli (Eds), *Educational perspectives on mathematics as semiosis: From thinking to interpreting to knowing* (pp.215–234). Brooklyn, NY & Ottawa, Ontario: Legas.

New South Wales Department of Education. (1989). *Language: Mathematics and language.* Sydney, NSW

New South Wales Department of School Education. (1997). *Teaching literacy in mathematics in Year 7.* Sydney, NSW

Quinnell, L. (2011). Cracking the code of NAPLAN numeracy tests in primary years: An exploration f the language used to assess numeracy. *Practically Primary*, 16(1), 19–21.

Quinnell, L., & Carter, L. (2011). Cracking the language code: NAPLAN numeracy tests in Years 7 and 9. Literacy *Learning: the Middle Years*, 19(1), 49–53.

Radford, L., & Puig, L. (2007). Syntax and meaning as sensuous, visual, historical forms of algebraic thinking. *Educational Studies in Mathematics*, 66(2), 145–164.

Rubenstein, R. N., & Thompson, D. R. (2001). Learning mathematical symbolism: Challenges and instructional strategies. *Mathematics Teacher*, 94(4), 265–271.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23(2), 139–159.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics* (7th ed.). Boston, MA: Pearson Education.

Zevenbergen, R., Dole, S., & Wright, R. (2004). *Teaching mathematics in primary schools.* Crows Nest, NSW: Allen & Unwin.